Short Papers

Effect of Discontinuities on the Group Delay of a Microwave Transmission Line

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Abstract—**The problem is considered of the effect of reflections from discontinuities at each end of a transmission line on the group delay at microwave frequencies. Previous work is briefly reviewed and a general analysis is made. Graphical data are presented based upon the formulas developed. Experimental results are given which confirm the theory.**

I. INTRODUCTION

The effects of mismatch on the calculation and measurement of group delay or envelope delay have been investigated in the past because of the importance of delay distortion in transmission systems. In recent experiments with space probes, small variations in the delay of microwave signals have been measured in order to obtain data on planetary atmospheres and the distribution of gaseous matter in space. Small errors in the determination of group delay have significant effects in these applications, and a need has arisen for a more thorough analysis to be carried out.

II. BACKGROUND

As Lewin *et al.* [1] explained in 1950, the effect of reflections from discontinuities is similar to the multipath problem [2]. A fraction of the main wave traveling down the transmission line is reflected back towards the source and rereflected there to combine with the main wave. In the case of small reflections, one can neglect multiple reflections and consider only the dominant effect. Since the reflected wave travels farther than the main wave, its phase varies faster with frequency. Consequently, the resultant phase shift ψ of the transmitted wave does not vary quite linearly with frequency, but has a cyclical variation superimposed on the linear variation. The group delay τ_g is related to the phase shift variation with frequency. At a frequency f_0, the group delay is defined as follows:

$$\tau_g = -\frac{1}{2\pi}\cdot\frac{d\psi}{df}\bigg|_{f=f_0} \tag{1}$$

Hence the group delay will also undergo cyclical variations due to reflections. Such an effect was observed by Lacy [3] in 1961.

Various analyses of this effect have appeared in the literature. In 1961, Lacy's [4] analysis was limited to the case of reactive shunt discontinuities on lossless transmission lines and did not yield an explicit expression for group delay. In 1964, Cohn and Weinhouse [5] gave a clear explanation of the effect and a simple analysis of the interaction phase error, stopping short of an explicit expression for group delay. In 1969, Drazy [6] gave an expression for the error in group delay due to reflections from test port terminations. His expression was valid for small reflections and lossless components but was limited in application. There may be other treatments of this problem in the literature, but the authors are not aware of them.

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III. THEORY

The following analysis is intended to be more general than previous work and will serve to clarify the assumptions made in calculating and measuring group delay of transmission lines. The analysis will be mainly useful for coaxial transmission-line problems but will be purposely kept general so as to include most uniconductor waveguide applications as well. It is not assumed that the discontinuities must be shunt susceptances, or that the characteristic impedance of the transmission line is identical to the characteristic impedances of the systems on either side. It is not assumed that the reflections from the discontinuities are small or equal. The discontinuities can be lossy or lossless, and need not obey the reciprocity condition.

Consider a *uniform* transmission line of length l having discontinuities at each end. It is assumed that l is long enough so that below cutoff higher order modes at and near the discontinuities do not cause appreciable interaction effects. A network model is shown in Fig. 1, in which the discontinuities are represented by the 2-ports L and N and the energy is assumed to propagate in the *dominant mode* from port 1 towards port 2.

The group delay τ_g is given by (1), where ψ is now ψ_{21}, the characteristic phase shift of the model, or the argument of S_{21}, its (transmission) scattering coefficient. If we employ conventional microwave network theory [7], we can calculate S_{21} for the three cascaded 2-ports and obtain

$$S_{21} = \frac{l_{21}n_{21}\exp(-\gamma l)}{1 - l_{22}n_{11}\exp(-2\gamma l)} \tag{2}$$

where $[\gamma = \alpha + j\beta]$ is the propagation constant of the transmission line and l_{21}, l_{22}, n_{11}, and n_{21} are scattering coefficients of the discontinuities. The attenuation and phase constants of the transmission line are α and β, respectively. For a general homogeneous transmission line such as a dominant mode uniconductor waveguide or coaxial line one can let [8, p. 262, eq. (8-52)]

$$\beta = \frac{2\pi}{\lambda_g} = \frac{2\pi}{\lambda_0}\left[\epsilon' - \left(\frac{\lambda_0}{\lambda_c}\right)^2\right]^{1/2} = \frac{2\pi f(\epsilon')^{1/2}}{c}\left[1 - \left(\frac{f_c}{f}\right)^2\right]^{1/2} \tag{3}$$

where λ_g is the waveguide wavelength, λ_0 is the free-space wavelength, λ_c is the cutoff wavelength, and ϵ' is the relative permittivity of the dielectric material filling the waveguide. It has been assumed that the relative permeability $\mu' = 1$ for the dielectric material. For TE and TM waves, λ_c is dependent on the internal dimensions of the waveguide cross section and is independent of ϵ'. For TEM waves λ_c is equal to infinity. The cutoff frequency f_c is related to the cutoff wavelength by the relationship

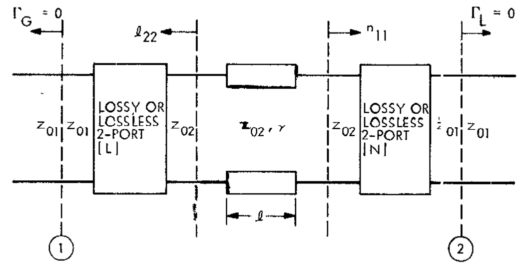

Fig. 1. Network model of transmission line and discontinuities. Γ_G is the voltage reflection coefficient of generator at port 1. Γ_L is the voltage coefficient of load at port 2.

$$f_c = c/[(\lambda_c)(\epsilon')^{1/2}]$$

where c is the velocity of EM waves *in vacuo*.

We can let

$$h = |l_{22}n_{11}| \exp(-2\alpha l)$$

$$\theta = 2\beta l - \arg(l_{22}) - \arg(n_{11})$$

and

$$\phi = \beta l - \arg(l_{21}) - \arg(n_{21}).$$

Then we can write

$$S_{21} \doteq \frac{|l_{21}n_{21}| \exp(-\alpha l) \cdot \exp(-j\phi)}{1 - h \exp(-j\theta)}. \quad (4)$$

The characteristic phase shift ψ_{21} is then

$$\psi_{21} = \arg(S_{21}) = -[\phi + \arg(1 - h \exp(-j\theta))]. \quad (5)$$

The group delay is obtained by multiplying (5) by -1 and differentiating the result with respect to angular frequency ($2\pi f$), and is

$$\tau_g = \tau_{gl} + \tau_{gn} + \tau_{g0} + \Delta\tau \quad (6)$$

where

$$\tau_{gl} = -(1/2\pi)(d/df) \arg(l_{21})$$

$$\tau_{gn} = -(1/2\pi)(d/df) \arg(n_{21})$$

and if v_g is the group velocity in the dielectric filled transmission line having phase constant given by (3)

$$\tau_{g0} = \frac{1}{2\pi}\left(\frac{d\beta}{df}\right) l = \frac{l}{v_g} = \frac{l}{c}\left(\frac{\lambda_g}{\lambda_0}\right)\epsilon'. \quad (7)$$

Furthermore

$$\Delta\tau = \frac{1}{2\pi}\,\frac{h(\cos\theta - h)(d\theta/df) + (dh/df)\sin\theta}{1 - 2h\cos\theta + h^2} \quad (8)$$

in which

$$\frac{1}{2\pi}\frac{d\theta}{df} = 2\tau_{g0} - \frac{1}{2\pi}\frac{d}{df}[\arg(l_{22}) + \arg(n_{11})] \quad (9)$$

and

$$\frac{1}{2\pi}\frac{dh}{df} = \frac{|l_{22}n_{11}|}{2\pi}\exp(-2\alpha l) \cdot \left[\frac{1}{|l_{22}|}\frac{d|l_{22}|}{df} + \frac{1}{|n_{11}|}\frac{d|n_{11}|}{df} - 2l\frac{d\alpha}{df}\right]. \quad (10)$$

We can see in (6)–(10) how a change with frequency of the attenuation or phase shift of the line or the reflection coefficients of the discontinuities, for example, might affect the group delay. In practical cases where the delays of the individual discontinuities are small, and we are interested in a relatively small bandwidth at microwave frequencies, we can neglect a number of terms that would contribute an insignificant amount to the final result. Specifically, if we neglect[1] τ_{gl}, τ_{gn}, and the term $(dh/df)\sin\theta$ in (8), then (6) simplifies to

$$\tau_g = \tau_{g0}\left[1 + \frac{2h(\cos\theta - h)}{1 - 2h\cos\theta + h^2}\right] = \tau_{g0} + \Delta\tau. \quad (11)$$

One can see that $\Delta\tau$ is a function of θ which varies with frequency, so that $\Delta\tau$ varies between the following limits:

$$-\frac{2\tau_{g0}h}{1+h} \le \Delta\tau \le \frac{2\tau_{g0}h}{1-h} \quad (12)$$

[1] For example, if the 2-ports L and N each correspond to shunt capacitances of 350 fF, then at a frequency of 8415 MHz, $\tau_{gl} = \tau_{gn} \simeq 0.007$ ns and $(1/2\pi)(dh/df)\sin\theta < 0.002$ ns. This corresponds to a practical case investigated experimentally and the results are presented later in this paper.

or from (7) and the definition of h preceding (4)

$$\frac{-2\left[\frac{l}{c}\left(\frac{\lambda_g}{\lambda_0}\right)\epsilon'\right][|l_{22}||n_{11}|\mathfrak{A}]}{1 + |l_{22}||n_{11}|\mathfrak{A}} \le \Delta\tau \le \frac{2\left[\frac{l}{c}\left(\frac{\lambda_g}{\lambda_0}\right)\epsilon'\right][|l_{22}||n_{11}|\mathfrak{A}]}{1 - |l_{22}||n_{11}|\mathfrak{A}} \quad (13)$$

where $\mathfrak{A} = \exp(-2\alpha l)$ is the attenuation (power) ratio of the length of transmission line which has an attenuation $A = -10 \cdot \log_{10}\mathfrak{A}$. The ratio (λ_g/λ_0) for a general homogeneous single-mode transmission line (including TEM-mode coaxial line) may be derived from (3). For an air–dielectric medium we may let $\epsilon' = 1$.

IV. GRAPHICAL DATA

A graph of the limits of $\Delta\tau$ is given in Fig. 2. It is assumed for simplicity that $|l_{22}| = |n_{11}|$, and that $\tau_{g0} = 100$ ns. If $|l_{22}| \neq |n_{11}|$, one can assume that the line has equivalent identical discontinuities at each end where the equivalent discontinuity has a reflection coefficient magnitude equal to $(|l_{22}||n_{11}|)^{1/2}$. For a transmission line having the same line attenuation but a τ_{g0} different from 100 ns, one multiplies the result obtained from the graph by the ratio of the actual τ_{g0} in nanoseconds to 100.

As an example of the use of Fig. 2, assume that a transmission line has a delay of 20 ns and a line attenuation of 5 dB prior to adding discontinuities of $|l_{22}| = 0.4$ and $|n_{11}| = 0.1$. Then an equivalent discontinuity placed at each end of this line would have a reflection coefficient magnitude of $[(0.4)(0.1)]^{1/2} = 0.2$. From Fig. 2, one finds that if the reflection coefficient magnitude of each discontinuity is 0.2 and the line attenuation is 5 dB for a 100-ns line, the limits of cyclical variation are ± 2.5 ns. Then the limits of cyclical variation for the above 20-ns line example are $\pm(20/100)(2.5)$ ns.

V. EXPERIMENTAL RESULTS

Experimental results were obtained to confirm the theory for the case of fairly large reflections. Metal disks having a diameter of 6.12 mm (0.241 in) and a thickness of 0.152 mm (0.006 in) were attached to the center conductors of short sections of 7-mm coaxial

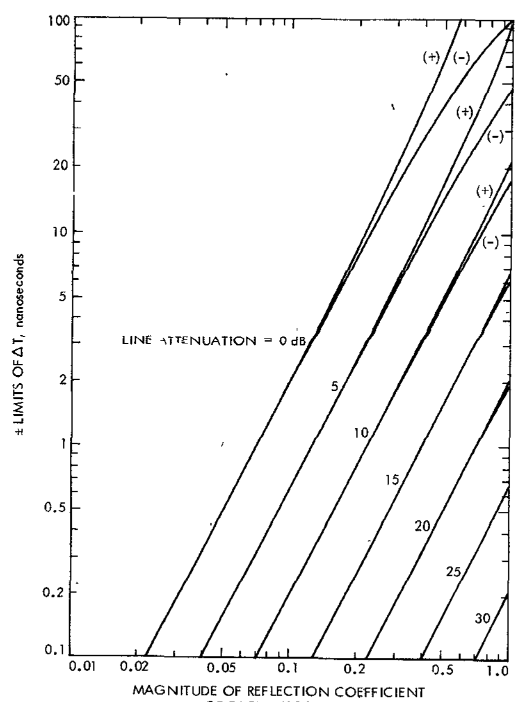

Fig. 2. Calculated limits of cyclical variation of group delay of a 100-ns transmission line with identical discontinuities at each end.

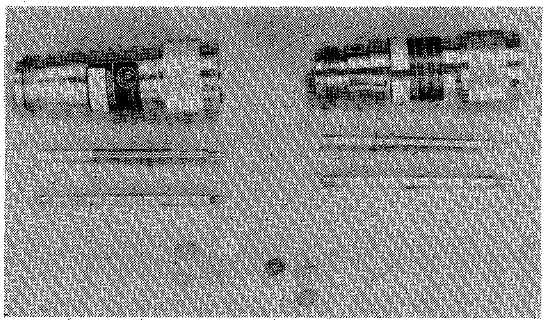

Fig. 3. 7-mm coaxial discontinuity assemblies.

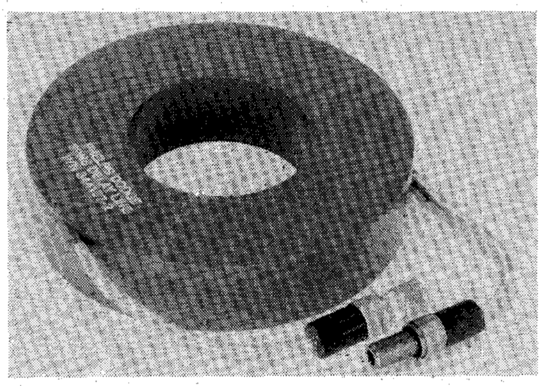

Fig. 4. Coaxial line delay standard of 30 ns.

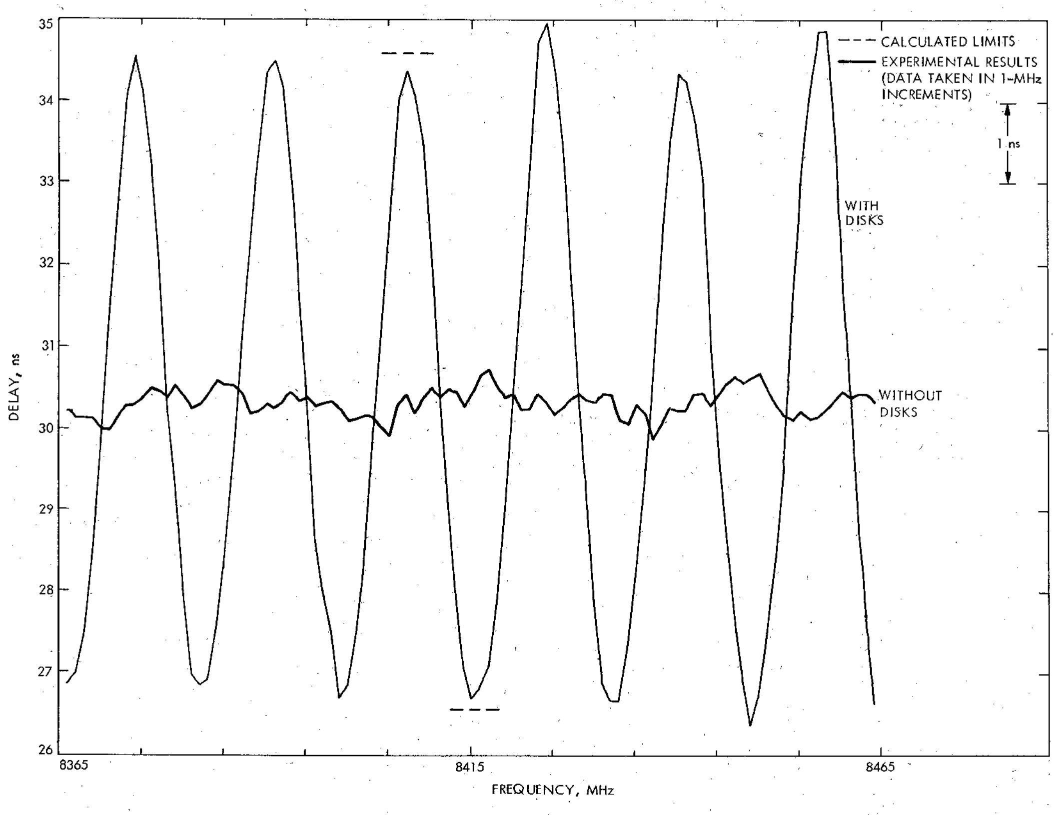

Fig. 5. Measured group delay of a 30-ns transmission line plus short end sections of 7-mm lines with and without 6.12-mm (0.241-in) diameter disks.

line to form the discontinuities L and N in Fig. 1. The discontinuity assemblies may be seen in Fig. 3. Over a frequency range of 8365–8465 MHz, a value of 0.42 for $|l_{22}|$ and $|n_{11}|$ of the disk assemblies was calculated using computer programs developed for 2-port standards [9]. Over the same frequency range a value of 0.43 was measured using an automatic network analyzer.

The 30-ns coaxial delay standard shown in Fig. 4 was connected between the discontinuities. Measurements of τ_g and of $-20 \log_{10} |S_{21}|$ using an automatic network analyzer are shown in Figs. 5 and 6, respectively. The group delay and the measured attenuation of $-10 \log_{10} \exp(-2\alpha l)$ of the transmission line plus the short sections of 7-mm coaxial line with disks removed are also shown. A comparison of calculated and measured results is shown in Table I. The tabulated results only show limits near the center of the frequency range but are typical of the results over the frequency range of 8365–8465 MHz. The calculated limits do not include the addi-

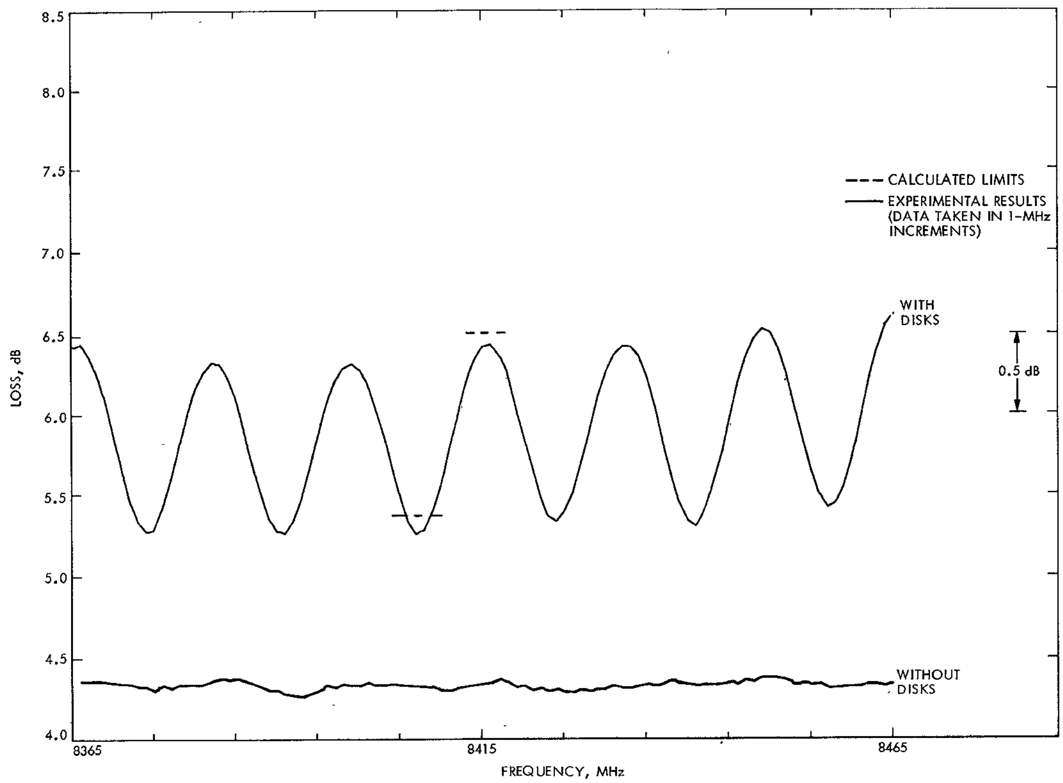

Fig. 6. Measured $-20 \log_{10} |S_{21}|$ of a 30-ns transmission line plus short end sections of 7-mm lines with and without 6.12-mm (0.241-in) diameter disks.

TABLE I
COMPARISON OF CALCULATED AND MEASURED UPPER AND LOWER LIMITS OF GROUP DELAY AND LOSS OF A 30-NS LINE WITH A DISCONTINUITY ASSEMBLY AT EACH END

Parameter	Calculated Value	Measured Value	Difference
Maximum Delay	34.60 ns	34.38 ns at 8407 MHz	0.22 ns
Minimum Delay	26.56 ns	26.70 ns at 8415 MHz	-0.14 ns
Maximum Loss	6.51 dB	6.42 dB at 8415 MHz	0.09 dB
Minimum Loss	5.36 dB	5.25 dB at 8407 MHz	0.11 dB

tional effects of reflections from the coaxial line connectors. Closer agreement was obtained by measuring the $|l_{22}|$ and $|n_{11}|$ taking into account the coaxial connectors.

Good agreement between theory and experiment was also obtained when additional measurements were made with different disks, different transmission-line lengths, and at an additional frequency range of 2235–2355 MHz. The details are not given here but will be included in a Jet Propulsion Laboratory report obtainable upon request to the authors.

VI. CONCLUSIONS

No attempt was made to experimentally confirm all aspects of the general theory. However, for the special case considered [identical discontinuities at each end, and negligible frequency dependence of α, $|l_{22}|$, $|n_{11}|$, arg (l_{21}), arg (n_{21}), arg (l_{22}), and arg (n_{11}) over the bandwidth of interest], adequate confirmation of the theory was obtained.

The analysis presented applies both to transmission lines operating in the TEM mode or to single-mode propagation in waveguides in general. The graphical data presented can be useful for the following: 1) estimating the limits on the variation of group delay with frequency or for 2) determining how much reduction of discontinuities is necessary in order to achieve a given accuracy in predicting group delay.

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Performance Characteristics of 4-Port Stripline Circulators

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***Abstract*—Impedance matrix formulation is applied to the 4-port stripline circulator to determine the optimum performance. The space harmonics generated in the ferrite disk are determined. The distribution of the electric field and the power densities across the ferrite disk are clearly presented. Using an inner dielectric provides more flexibility in the design.**

I. INTRODUCTION

The circulation adjustment of 4-port circulators cannot be reduced to an admittance matching procedure as in the 3-port case [1]. Three circulation conditions must be satisfied. Davies and Cohen [2] derived three equations describing the circulation conditions of a 4-port stripline circulator. Recently, Ku and Wu [3] have solved this problem by using Bosma's Green function method [4]. Several experimental studies were reported together with phenomenological description using the scattering matrix [1], [5], [6].

Here we use the impedance matrix formulation [7]–[9] to determine the circulation conditions and performance characteristics. To have a better insight into the circulation mechanism of the 4-port circulator, the space harmonics generated in the ferrite disk are determined. The field amplitude and the distribution of power densities across the ferrite disk are calculated. The effect of introducing an inner dielectric on the circulation conditions and performance is studied.

II. CIRCULATOR CHARACTERISTICS

Considering the boundary conditions for a symmetrical 4-port ferrite junction, the impedance matrix can be written in the form

$$\begin{bmatrix} E_1 \\ E_2 \\ E_3 \\ E_4 \end{bmatrix} = \begin{bmatrix} Z_1 & Z_2 & Z_3 & Z_4 \\ Z_4 & Z_1 & Z_2 & Z_3 \\ Z_3 & Z_4 & Z_1 & Z_2 \\ Z_2 & Z_3 & Z_4 & Z_1 \end{bmatrix} \begin{bmatrix} H_1 \\ H_2 \\ H_3 \\ H_4 \end{bmatrix} \quad (1)$$

where

$$Z_1 = jL \quad (2)$$

$$Z_2 = jM - K \quad (3)$$

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TABLE I
THE EFFECT OF VARYING ψ ON THE CIRCULATION CONDITION AND RELATIVE BANDWIDTHS

ψ°	k/μ	x	Z_e/Z_d	B.W. % 1-3	B.W. % 1-4
5	0.3166	3.795	1.0525	2.05	6.60
10	0.3105	3.780	0.5320	2.15	6.94
15	0.3055	3.759	0.3740	2.29	7.06

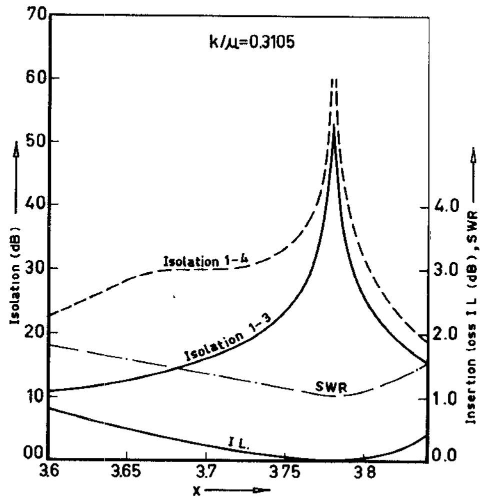

Fig. 1. Performance characteristics of a 4-port circulator. $k/\mu = 0.3105$, $Z_e/Z_d = 0.532$, and $\psi = 10°$.

$$Z_3 = jN \quad (4)$$

$$Z_4 = jM + K \quad (5)$$

and where E_1, E_2, E_3, and E_4 are the average values of the electric field at the four ports. The corresponding assumed constant values of the azimuthal component of the magnetic field are H_1, H_2, H_3, and H_4. K, L, M, and N are defined in [2]. Impedance matrix formulation has the advantage that it can handle easily externally tuned circulators [9].

Imposing the circulation condition (the ports 3 and 4 are completely isolated, and the input impedance is purely resistive and equals the wave impedance of the dielectric filling the striplines Z_d) on (1), three equations can be obtained [2]. These equations are solved considering up to the twelfth space harmonic. The computational results are given in Table I.[1] The table shows that as the stripline coupling angle ψ increases both the normalized radius x and the ratio k/μ decrease slightly. This is accompanied by a large decrease in the impedance ratio Z_e/Z_d. These results agree favorably with Ku and Wu [3].

The performance characteristics [isolation at ports 3 and 4, insertion loss, and standing-wave ratio (SWR)] are computed for $\psi = 10°$, $k/\mu = 0.3105$, and $Z_e/Z_d = 0.532$ (Fig. 1). At $x = 3.78$ we have an ideal circulation (infinite isolation, zero insertion loss, and unity SWR). The relative 20-dB bandwidth for a 4-port circulator can be defined as the ratio of the band over which the isolation at ports 3 or 4 exceeds 20 dB to the central circulation frequency. Increasing ψ results in a small increase in the bandwidth at ports 3 and 4 (Table I). It is also shown that the isolation bandwidth at port 3 is less than that at port 4.

[1] The parameters ψ, k, μ, x, Z_e, and Z_d are defined in [2].